利用兩層斜壓模式探討對流系統的發展與減弱

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1.Introduction

There are two kinds condition which related to the weather system whether intensify or weaken. One is dynamic condition; the other is thermal condition for a convective system. In a benefit situation the system will self-development. But in a non- benefit situation the system will be self-weakness.

In this research we use two level model to study the diabatic effect and terrain effect for the development of weather system. From the result of our study, the intensification or weakness of the convective system can be explained clearly.

2.The governor equations

For a mesoscale system, we apply thermal dynamic equation and hydrostatic equation on two- layer model to study the adjustment process and the equations are given by:

Thermal dynamic equation

$$\mathrm{D}\, ln\theta / \mathrm{d}t = \dot{\mathrm{Q}} / \mathrm{c}_p T \quad \text{-----}(1.1)$$

hydrostatic equation

$$\partial\Phi/\partial p = -RT/p \quad \text{-----}(1.2)$$

With equation (3.1) and (3.2) yields

$$\partial(\partial\Phi/\partial p)/\partial t + \mathrm{u}\partial(\partial\Phi/\partial p)/\partial \mathrm{x} + \mathrm{v}\partial(\partial\Phi/\partial p)/\partial \mathrm{y} + (\mathrm{c}_a^2/\mathrm{p}^2)\omega = -R\dot{\mathrm{Q}}/p\mathrm{c}_p \quad \text{-----}(1.3)$$

c_a^2 ：Static stability parameters, $\mathrm{c}_a^2 = R^2 T / g(\gamma_d - \gamma)$

γ_d, γ ： Dry adiabatic lapse rate and lapse rate of temperature

c_p ：Specific heat of dry air at constant pressure

$\dot{\mathrm{Q}}$ ：Diabetic heating rate

As the weather system departs from geostrophic balance, this can be lead to excitement in inertia gravity waves, which will adjust the mass field and momentum field to return to geostrophic balance. If the system is in a small scale, for simplicity, we suppose the motion of atmosphere is adiabatic with constant Coriolis parameter (f_0). We also consider the atmosphere is linearized motion about a basic state of motionless. With the perturbation method, the horizontal momentum and thermal energy equations are written by

$$\partial u'/\partial t - f_0 v' = -\partial\Phi'/\partial \mathrm{x} \quad \text{-----}(1.4)$$

$$\partial v'/\partial t + f_0 u' = -\partial \Phi'/\partial y \quad \text{--}(1.5)$$

$$\partial(\partial \Phi'/\partial p)/\partial t + (c_a^2/p^2)\omega' = 0 \quad \text{--}(1.6)$$

(′) which means perturbation.

Taking $\partial(1.4)/\partial x + \partial(1.5)/\partial y$ and then dropping (′) yields

$$\partial^2 u/\partial x\partial t + \partial^2 v/\partial y\partial t - f_0(\partial v/\partial x - \partial u/\partial y) = -\partial^2 \Phi/\partial x^2 - \partial^2 \Phi/\partial y^2$$

$$\partial(\partial u/\partial x + \partial v/\partial y)/\partial t - f_0(\partial v/\partial x - \partial u/\partial y) = -\partial^2 \Phi/\partial x^2 - \partial^2 \Phi/\partial y^2$$

$$\partial D/\partial t - f_0 \zeta = -\nabla^2 \Phi \quad \text{--}(1.7)$$

Where

$D = \partial u/\partial x + \partial v/\partial y$

$\zeta = \partial v/\partial x - \partial u/\partial y$

Taking $\partial(3.5)/\partial x - \partial(3.4)/\partial y$ and dropping (′) then yield

$$\partial \zeta/\partial t + f_0 D = 0 \quad \text{--}(1.8)$$

Dropping (′) from equation (3.6) then yield

$$\partial(\partial \Phi/\partial p)/\partial t + (c_a^2/p^2)\omega = 0 \quad \text{-- --}(1.9)$$

Equation (1.7) is the divergent equation. Equation (1.8) is the vorticity equation. Equation (3.9) is the thermal dynamic equation with vertical motion. With equation (1.7), (1.8) and (1.9), we set up a two layer model as shown in figure 1. We use the following equations on level 1 and level 3 to study the adjustment process associated with gravity wave.

$$\partial D/\partial t = f_0 \zeta - \nabla^2 \Phi \quad \text{--}(1.10)$$

$$\partial \zeta/\partial t = -f_0 D \quad \text{--}(1.11)$$

$$\partial(\partial \Phi/\partial p)/\partial t = -(c_a^2/p^2)\omega \quad \text{--}(1.12)$$

or $\quad \partial \Phi/\partial t = (c_a^2/p)\omega \quad \text{--}(1.13)$

With the above equation we discuss the lateen heat release and terrain effect to the development of a system.

level 0	0 P	————————————	Φ_0, p_0
level 1	1/4 P	-----------ζ_1-----D_1------Φ_1---------------	Φ_1, p_1
level 2	1/2 P	————————ω_2————	Φ_2, p_2
level 3	3/4 P	-----------ζ_3-----D_3-----Φ_3--------------	Φ_3, p_3
level 4	P	————————————	Φ_4, p_4

Figure 1 The variable in the vertical columns for two-level model.

On figure12, the capital P is the pressure of the bottom level, we suppose P is constant. And ω_2 is the vertical motion at level 2. Φ is the geopotential height at the top level. The continuity equation can be expressed as $D = -\partial\omega/\partial p$.

At the boundary, the vertical motion at both the top level (ω_0) and the bottom level (ω_4) are all equal zero.

3.The diabatic heating effect

In two level model, the variable of thermal dynamic equation in level 2 can be written

$\partial T_2/\partial t + V_2 \cdot \bigtriangledown\ T_2 - \sigma\omega_2 = \dot{Q}/C_p$

$\partial T_2/\partial t + V_2 \cdot \bigtriangledown\ T_2 = \sigma\omega_2 + \dot{Q}/C_p$

If the basis state is no motion then V_2 is zero. And the hydrostatic equation in level 2 is written by

$T_2 = -(p_2/R)(\Phi_1 - \Phi_3)/(p_1 - p_3) = +(p_2/R)(2\Delta\Phi/\Delta p)$,

Where $(p_1 - p_3) = -P/2$, $(\Phi_1 - \Phi_3) = \Delta\Phi$, $p_2 = (1/2)P$, P is the pressure of bottom layer.

$\partial T_2/\partial t = +\partial (p_2/R)(2\Delta\Phi/\Delta p)/\partial t = (1/R)(\partial\Delta\Phi/\partial t)$

$(1/R)(\partial\Delta\Phi/\partial t) = \sigma\omega_2 + \dot{Q}/C_p$

$\omega_2 = (1/\sigma R)(\partial\Delta\Phi/\partial t) - \dot{Q}/(\sigma C_p))$

From vorticity equation

$\partial\zeta/\partial t = -V \cdot \bigtriangledown(\zeta + f) + f_0\partial\omega/\partial p$

With finite difference, both the bottom and upper layer where ω_0, ω_4 are equal zero, then

$\partial\omega_1/\partial p = (\omega_0 - \omega_2)/(p_0 - p_2) = +2\omega_2/P$ where $(p_0 - p_2) = 0 - (1/2)P = -(1/2)P$, $\omega_0 = 0$

$\partial\omega_3/\partial p = (\omega_2 - \omega_4)/(p_2 - p_4) = -2\omega_2/P$ where $\Delta p = (p_2 - p_4) = (1/2)P - P = -(1/2)P$, $\omega_4 = 0$

and

$\partial\zeta_1/\partial t = -V_1\bigtriangledown(\zeta_1 + f) + 2f_0\omega_2/P$

$\partial\zeta_3/\partial t = -V_3\bigtriangledown(\zeta_3 + f) - 2f_0\omega_2/P$

If the basis state is no motion V_1, V_2 and V_3 all are zero.

Then

$\partial\zeta_1/\partial t = 2f_0\omega_2/P = (2f_0/\sigma PR)\,(\partial\Delta\Phi/\partial t - \dot{Q}/(\sigma C_p))$

$\partial\zeta_3/\partial t = -2f_0\omega_2/P = (2f_0/\sigma PR)\,(\dot{Q}/(\sigma C_p) - \partial\Delta\Phi/\partial t)$

That imply

$\partial\zeta_1/\partial t \;\; \alpha \;\; -\dot{Q}/(\sigma C_p)$ and also $\partial\zeta_1/\partial t \;\; \alpha \;\; \partial\Delta\Phi/\partial t$ where $\Delta\Phi = \Phi_0 - \Phi_2$

$\partial\zeta_3/\partial t \;\; \alpha \;\; \dot{Q}/(\sigma C_p)$ and also $\partial\zeta_3/\partial t \;\; \alpha - \;\; \partial\Delta\Phi/\partial t$ where $\Delta\Phi = \Phi_2 - \Phi_4$

From above relation, the vorticity is associated with time change of geopotential high and diabatic heating. At low level the vorticity is increased by diabatic heating, but at high level the vorticity is reduced by diabatic heating such as figure 2.

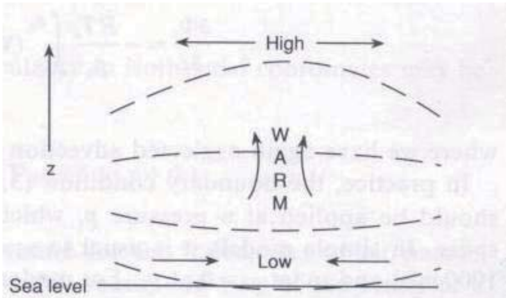

figure 2 The development of convective system (From Holton 1992). Dashed lines indicate isobars.

If there is friction in the boundary, the Ekman pumping effect,

will cause the horizontal wind toward low pressure. A secondary circulation occurs in a rotating fluid. And at the top of the boundary the vertical motion can be gotten as

$\omega = -\rho g\zeta(K/2f)^{1/2}$

From thermodynamic equation

$\partial T/\partial t + V \cdot \nabla T - \sigma\omega = \dot{Q}/C_p$

$\sigma\omega = -(\dot{Q}/C_p + \partial T/\partial t + V \cdot \nabla T)$

$\sigma\omega_3 = -(\dot{Q}/C_p + \partial T_3/\partial t + V_3 \cdot \nabla T_3)$

$\dot{Q}/C_p = -\sigma\omega_3 + \partial T_3/\partial t + V_3 \cdot \nabla T_3$

If the basis state is no motion then V_3 is equal to zero.

$\dot{Q}/C_p = -\sigma\omega_3 + \partial T_3/\partial t$

$\partial T_3/\partial t = \dot{Q}/C_p - \sigma\omega_3$

Now we can also assume the bottom vertical motion is $\omega_4=0$.Then the finite difference of $\partial\omega_3/\partial p$ is equal to $(\omega_2 - \omega_4)/(p_2 - p_4) = -\omega_2/2P$. Above the bottom level 4 is the level 3. So we assign the top of the boundary layer on level 3. And $\omega_3 = -\rho g\zeta_3(K/2f)^{1/2}$

From hydrostatic equation

$-\partial\Phi/\partial p = (R/p)T$

$2\Delta\Phi/\Delta p = (R/p_3)T_3$ where $\Delta\Phi = \Phi_2 - \Phi_4$

Because of Ekman pumping effects likes initiate forces that makes the bottom layer vertical motion and bring the vapor upward motion. If there is absence of convective cloud the spin-down process will destroying the vorticity intensity. As the vapor up to the upper layer the latent release of condensation $\dot{Q}$ warm the air column.

The vertical motion of bottom layer induces the bottom layer convergence and upper divergence. And the convergence associate with the vorticity $\partial\zeta_3/\partial t$ increase. As ζ_3 increase then $\omega_3 = -\rho g\zeta_3(K/2f)^{1/2}$ is also increase that could cause a recycle process. Hence the development of the vorticity of bottom layer is increased definitely.

4.The terrain effect

From continuity equation

$\partial u/\partial x + \partial v/\partial y = -\partial\omega/\partial p$

Let divergence $D = \partial u/\partial x + \partial v/\partial y = -\partial\omega/\partial p$

Then in level 3 we can get vorticity equation as follow

$\partial\zeta_3/\partial t = +f_0(\omega_2 - \omega_4)/(p_2 - p_4) = -2f_0(\omega_2)/P = (2f_0/\sigma RP)(\dot{Q}/(\sigma C_p) - \partial\Delta\Phi/\partial t)$

The vertical motion (W_s) induced by terrain effect as follow

$W_s = u_s\partial h_s/\partial x + vs\partial h_s/\partial y$

$\omega_3 = -\rho g W_s = -\rho g(u_s\partial h_s/\partial x + vs\partial h_s/\partial y)$

from vorticity equatiom

$\partial\zeta/\partial t = -V\cdot\nabla(\zeta+f) + f_0\partial\omega/\partial p = -V\cdot\nabla(\zeta+f) + f_0\partial\omega/\partial p$

$= -V\cdot\nabla(\zeta+f) - f_0(\partial u/\partial x + \partial v/\partial y)$

Then $\partial\zeta/\partial t = = -V\cdot\nabla(\zeta+f) - f_0(\partial u/\partial x + \partial v/\partial y) = -V\cdot\nabla(\zeta+f) - f_0 D$

If the environment is no basic flow then $V = 0$ and there is only perturbation system on the air. then

$\partial\zeta/\partial t = -f_0 D$

$\partial\zeta_3/\partial t = -f_0 D_3$

$\partial T/\partial t + V\cdot\nabla T - \sigma\omega = \dot{Q}/C_p$

From equation (1.12)

$\partial\Delta\Phi/\partial t = 2(c_a^2/p)\omega_3$ where $\Delta\Phi = \Phi_2 - \Phi_4$

$\omega_3 = -\rho g W_s = -\rho g(u_s\partial h_s/\partial x + vs\partial h_s/\partial y)$

$\partial\Delta\Phi/\partial t = -2(c_a^2/p)\rho g(u_s\partial h_s/\partial x + vs\partial h_s/\partial y)$

If there is only $\partial h_s/\partial x$ where we suppose $\partial h_s/\partial y = 0$ such as figure 3.

$\partial\Delta\Phi/\partial t = -2(c_a^2/p)\rho g(u_s\partial h_s/\partial x)$

At the westside of the mountain

$\partial h_s/\partial x > 0 \quad \omega_3 < 0$

$\partial T/\partial t + V\cdot\nabla T - \sigma\omega = \dot{Q}/C_p$

If there is no condensation process ($\dot{Q}/C_p = 0$) and no basic mean flow ($V = 0$) then the thermal equation as follow

$\partial T/\partial t - \sigma\omega = 0 \quad \partial T_3\partial t = \sigma\omega_3$

The vertical upward motion causes the bottom of the air column cooling and shrinks then cause convergence at the bottom and $\partial\Delta\Phi/\partial t < 0$

The terrain effect that makes the flow upward motion causes the air column cooling associated with the low level vorticity increase.

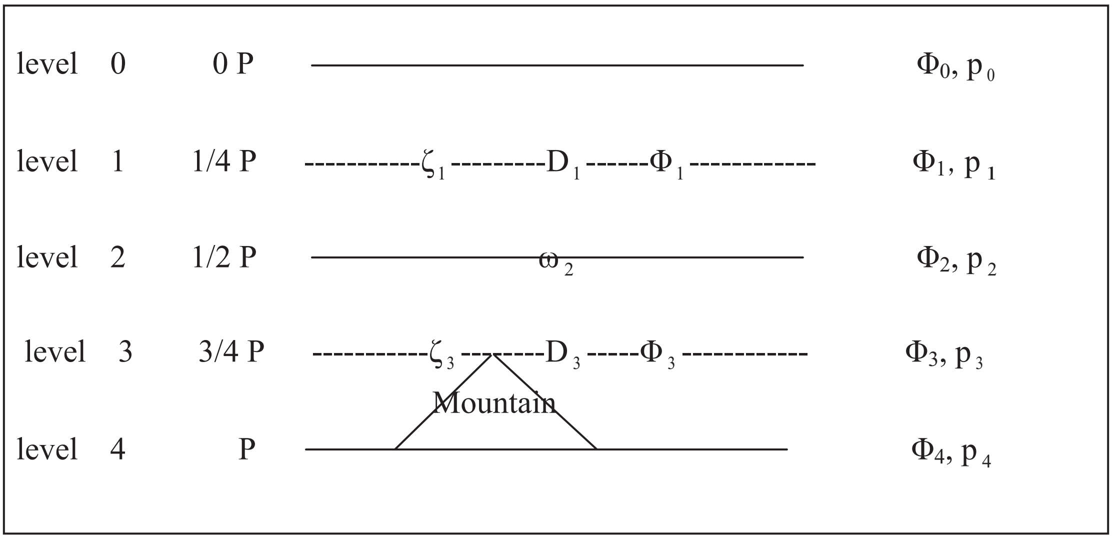

Figure 3 The variable in the vertical columns for two-level model with mountain from bottom layer to level 3.

5.Conclusion

As a result, from two level model we can prove the lateen heat release of convective system and terrain effect could benefit for the system development.

6.Reference

Holton, J. R. 1992：An introduction to dynamic meteorology. Academic Press.