Comparison of hybrid and generalized hybrid vertical coordinates implemented in the NCEP GFS

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Abstract

It is common to use hybrid vertical coordinates in atmospheric and oceanic modeling. The combination of different coordinates into a hybrid coordinates system can take advantage of the strengths of the individual types of coordinates surfaces for numerical purposes. In the NCEP Global Forecast System(GFS), hybrid vertical coordinate of sigma-pressure is applied for operation. A specified definition of a generalized hybrid vertical coordinate, including sigma, pressure and isentropic s urface had been implemented in the NCEP GFS (Juang, 2005). With more flexibility and less approximations, generalized hybrid vertical coordinate should provide a better dynamic field to improve weather and climate predictions.

In this study, we use operational NCEP GFS daily runs as control case, compare results with sigma-pressure and sigma-theta in generalized hybrid vertical coordinates to evaluate if generalized vertical coordinate provide improvement. Results from a period of daily forecasts up to 5 days were collected. The anomaly correlations of these two generalized hybrid coordinates show either better or the same level of skills compared to the NCEP operational GFS, but root mean square and bias are mixed.

Key word: generalized hybrid coordinate

1. Introduction

It has became a trend to use hybrid vertical coordinates in atmospheric modeling (Simmons and Burridge 1981; Zhu et al 1992; Konor and Arakawa 1997; Johnson and Yuan 1998; Benjamin et al 2004). With hybrid coordinates, the atmospheric model can be integrated along different types of coordinates surfaces. The coordinates near the surface and lower atmosphere still use terrain-following sigma coordinates, but over the upper atmosphere better results come from computations on quasi-horizontal coordinates, such as pressure surfaces or isentropic surfaces, that reduce the numerical errors due to vertical motion calculations. The combination of these coordinates into a hybrid coordinate system can take advantage of the strengths of the individual types of coordinate surfaces for numerical purposes.

Hybrid vertical coordinate of sigma-pressure is applied in the NCEP operational Global Forecast System(GFS) for many years. A specified definition of a generalized hybrid vertical coordinate, including sigma, pressure and isentropic surface had been implemented in the NCEP GFS (Juang, 2005). In this study, we use operational NCEP GFS daily runs as control case, compare results with sigma-pressure and sigma-theta in generalized hybrid vertical coordinates to evaluate if generalized vertical coordinate provide improvement.

In this paper, section 2 illustrates primary differences between sigma-pressure hybrid coordinate in operational GFS and generalized hybrid coordinate; section 3 gives some model results from different vertical coordinates. Discussion and conclusion are in section4.

2. Generalized Hybrid Coordinate

A discretization of a hydrostatic primitive equation global atmospheric model on spherical and generalized vertical coordinates is described in NCEP Office Notes 445 (Juang 2005). The discretization in the horizontal using a spectral method with spherical transformation is as the same as used in NCEP global model.

Energy and angular momentum conservation are used as constraints to discretize the vertical integration by finite difference scheme. The entire atmosphere is divided to several layers, the vertical grid structure is shown in Fig.1, with the lowest layer as 1, top layer as the Kth layer and K+1st interface, any middle layer as k. Only pressure and vertical flux are specified at the interfaces, and other variables such as horizontal wind, temperature, specific humidity and specific amount of tracers are specified at each layer. Conservation is a constraint that requires the pressure at each layer to be averaged by the pressures at immediate neighbor interfaces(the one above and one below a given layer). Since pressures are not combined from a pressure gradient and density in a logarithmic form, the relationship for pressure between layers and interfaces becomes simple, and with pressure equation not in logarithmic, it provides mass conservation extra. The detailed derivations in differential form of generalized coordinate can be found in NCEP Office Note #445 (Juang 2005).

The primary differences between sigma-pressure hybrid coordinate in operational GFS and generalized hybrid coordinate are vertical discretization.

First, in generalized hybrid coordinate, we can write surface pressure equation as surface form after discretization:

$$\frac{\partial p_S}{\partial t}=-m^2\sum_{i=1}^{K}\left((\hat{p}_i-\hat{p}_{i+1})\left(\frac{\partial u_i^*}{a\partial\lambda}+\frac{\partial v_i^*}{a\partial\varphi}\right)+u_i^*\frac{\partial(\hat{p}_i-\hat{p}_{i+1})}{a\partial\lambda}+v_i^*\frac{\partial(\hat{p}_i-\hat{p}_{i+1})}{a\partial\varphi}\right) \tag{2.1}$$

In operational GFS, surface pressure is transferred to logarithm form, and surface pressure equation is written as

$$\frac{\partial \ln Ps}{\partial t}=-\frac{1}{Ps}\sum_{k=1}^{levs}(\nabla\cdot\upsilon_{Hk}\Delta_{Pk}),\ \Delta p_K=p_{K+\frac{1}{2}}-p_{k-\frac{1}{2}}>0 \tag{2.2}$$

Second, the way to define geopotential height and pressure at layer in hydrostatic equation is different in generalized hybrid coordinate and operational GFS. In generalized coordinate discretization, pressure is provided at levels as shown in Fig.1, but we still need pressure at layers to calculate/conserve angular momentum equation, define $p_k=f(\hat{p}_{k+1},\hat{p}_k)$, so

$$\nabla p_k=\frac{\partial p_k}{\partial\hat{p}_{k+1}}\nabla\hat{p}_{k+1}+\frac{\partial p_k}{\partial\hat{p}_k}\nabla\hat{p}_k \tag{2.3}$$

and put this definition into equation to satisfy energy conservation, we can obtain

$$\frac{\partial p_k}{\partial\hat{p}_{k+1}}=\frac{\partial p_k}{\partial\hat{p}_k}=\frac{1}{2} \tag{2.4}$$

Therefore,

$$p_k=\frac{1}{2}(\hat{p}_{k+1}+\hat{p}_k) \tag{2.5}$$

it is a simple solution for the function of p at layers.

In operational GFS, the form chosen for the finite-difference analog of the hydrostatic equation is

$$\phi_{k+1/2}-\phi_{k-1/2}=-RT_k\ln\frac{p_{k+1/2}}{p_{k-1/2}}, \tag{2.6}$$

where $\phi_{k+1/2}$ is a half-level value. When used in the momentum equations full-level (layer) values of ϕ are required as

$$\phi_k=\phi_{k+\frac{1}{2}}+\alpha_k R(T_v)_k \tag{2.7}$$

In order to preserve the conservation of angular momentum, the definition of α_k as

$$\alpha_k=1-\frac{p_{k-1/2}}{\Delta p_k}\ln\frac{p_{k+1/2}}{p_{k-1/2}},\ \text{for } k\ge 1 \tag{2.8}$$

Furthermore, we have upper boundary condition as α_1=ln2. And pressure at full-level is expressed as

$$p_k=\frac{\Delta p_k}{\ln(p_{k+1/2}/p_{k-1/2})}, k>1 \tag{2.9}$$

$$p_1=1/2\Delta p_1$$

This way may have higher accuracy but need predefinition to satisfy all equations.

A generalized vertical coordinate used in generalized coordinate GFS is shown as

$$\hat{p}_k=\hat{A}_k+\hat{B}_k p_s+\hat{C}_k\left(\hat{T}_{vk}/\hat{T}_{0k}\right)^{C_p/R_d} \tag{2. 10}$$

which can be used for sigma, sigma-pressure, sigma-theta, and sigma-theta-pressure, etc. At any given model layers, all of A, B, and C have to fix to constants. While A=C=0, it is pure sigma coordinate. While either A or C is non-zero, it is hybrid coordinate. With generalized vertical coordinate, we can define different hybrid vertical coordinate by our choice. This is a flexible way to provide vertical coordinate in an atmospheric model for more research purposes.

3.Case Results

In order to get a sense of how well the generalized hybrid vertical coordinates perform in GFS, we set operational GFS as control run to compare with generalized hybrid coordinate and generalized hybrid coordinate with enthalpy as thermodynamic variable. The results are plotted together with the results from the operational GFS, which is a sigma-pressure hybrid coordinate based on discretization of Simmons and Burridge (1981), as previously mentioned.

Fig.2 shows the anomaly correlation of geopotential height as 500 hPa after 5 days integration over Northern Hemisphere from January to March 2010 for the operational GFS, generalized hybrid coordinate (PRGW) , and generalized hybrid coordinate with enthalpy (PRGEW). Since all of these runs have similar numerics, resolution (T382L64), and physics, their performances follow each other on a day-by-day basis, but generalized hybrid coordinates cases have slightly higher anomaly correlation score than GFS. Fig. 3 and Fig. 4 show the same as Fig. 2, except that Fig. 3 is for Southern Hemisphere and Fig. 4 is for Tropical zone. The Southern Hemisphere and Tropical zone performance are similar to the Northern Hemisphere; they follow each other on a day-by-day basis and show the same level of anomaly correlation score.

In addition to the anomaly correlation, generalized hybrid coordinates cases show lower Root Mean Square (RMS) error than operational GFS. Fig. 5 shows the Root Mean Square (RMS) error of Northern Hemisphere geopotential height at 850 hPa after 5-day integration, from January to March 2010. The RMS errors from generalized coordinates are consistently smaller than operational GFS near every day. Fig. 6 shows the Root Mean Square (RMS) error of Northern Hemisphere vector wind at 850 hPa after 5-day integration, from January to March 2010. The RMS errors from generalized coordinates are consistently smaller than operational GFS near every day.

In Fig. 7, upper left panel shows mean of RMS errors of Northern Hemisphere wind vector and lower left one and upper right one show the difference of mean RMS error between generalized coordinate and operational GFS and the difference of mean RMS error between generalized coordinate with enthalpy and operational GFS. Mean RMS error of generalized coordinate is lower than operational GFS, but PRGEW is not. This result may come from physic part in GFS, because physics should be modified when using enthalpy as thermodynamic variable.

4.Conclusion

The results from the comparison of three different coordinate systems show very similar scores with resolution T384L64. All results indicate generalized hybrid vertical coordinates have successfully implemented in GFS. In summary, generalized vertical coordinate has better performance than hybrid vertical coordinate in operational GFS. With more flexibility and less approximation, generalized hybrid vertical coordinate should provide a better dynamic field to improve numerical weather and climate predictions.

5. Reference

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6. Figures

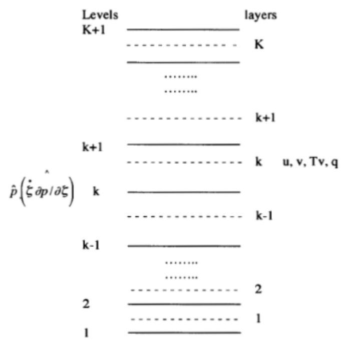

Fig.1 The vertical grid structure with layers and levels. Integers are used to index layers and levels; variables marked with hats are on levels and without hats are layers.

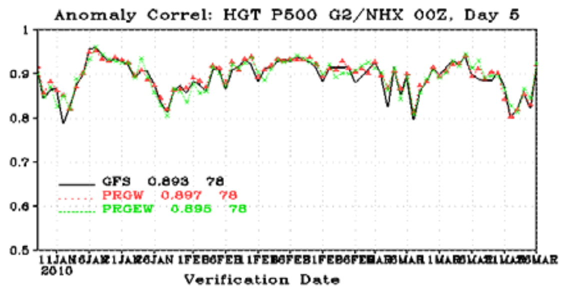

Fig. 2 Anomaly correlation for the Northern Hemisphere at 500 hPa after 5 days integration for cases from January to Mar 2010 with the operational GFS using sigma-pressure hybrid coordinate (GFS) and modified GFS using generalized hybrid coordinates (PRGW) and generalized hybrid coordinates with enthalpy as thermodynamic variable (PRGEW), with resolution of T382 and 64 layers.

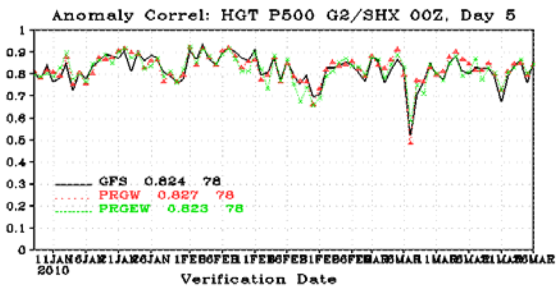

Fig. 3 The same as Fig.1, but for Southern Hemisphere.

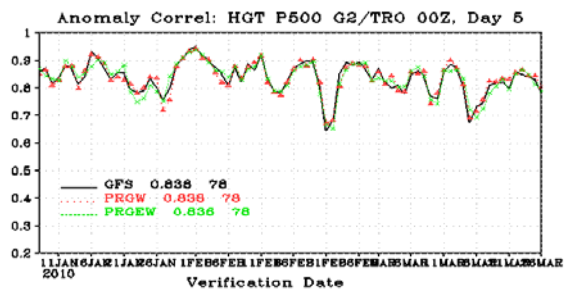

Fig. 4 the same as Fig. 2, but for Tropical zone.

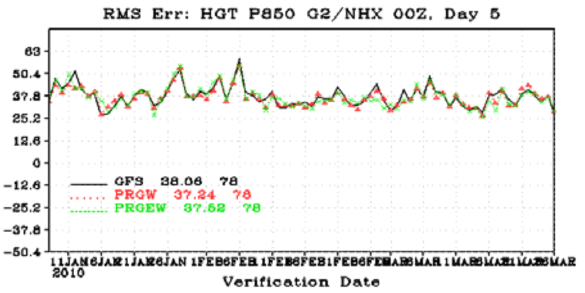

Fig. 5 Root Mean Square error of Northern Hemisphere geopotential height at 850 hPa after 5-day integration for cases from January to March 2010 with operational GFS using sigma-pressure hybrid coordinate (GFS) and modified GFS using generalized hybrid coordinates (PRGW) and generalized hybrid coordinates with enthalpy as thermodynamic variable (PRGEW), with resolution of T382 and 64 layers.

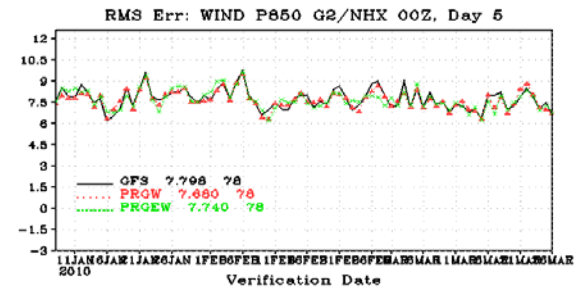

Fig. 6 Root Mean Square error of Northern Hemisphere vector wind at 850 hPa after 5-day integration for cases from January to March 2010 with operational GFS using sigma-pressure hybrid coordinate (GFS) and modified GFS using generalized hybrid coordinates (PRGW) and generalized hybrid coordinates with enthalpy as thermodynamic variable (PRGEW), with resolution of T382 and 64 layers.

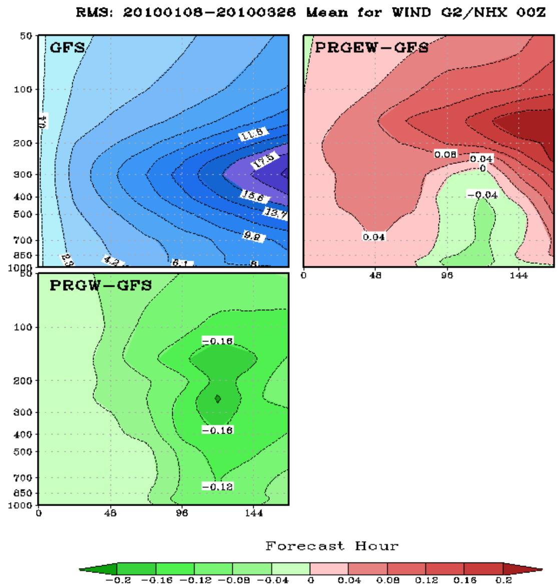

Fig. 7 Mean of Root Mean Square error of Northern Hemisphere vector wind for cases from January to March 2010 with operational GFS using sigma-pressure hybrid coordinate (GFS) and modified GFS using generalized hybrid coordinates (PRGW) and generalized hybrid coordinates with enthalpy as thermodynamic variable (PRGEW), with resolution of T382 and 64 layers.