A MASS-CONSERVING POSITIVE-DEFINITE SEMI-LAGRANGIAN ADVECTION FOR SEDIMENTATION OF PRECIPITATION

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1. Introduction

Most numerical weather prediction (NWP) models employ a bulk-type cloud microphysics scheme to account for grid-resolvable precipitation processes. The bulk-type cloud microphysics is one of time-demanding components in the NWP model, even though it is cheaper than the bin-type scheme. As compared to the continuous efforts given to the development of the advanced microphysical processes in cloud modeling area, a few efforts have been given to the improvement of the accuracy in computing sedimentation process for precipitating particles in the cloud modeling area in atmospheric models. Most NWPs in computing the sedimentation of precipitation, almost all the cloud schemes use a sub-time step so that falling particles do not cross the vertical grid within the model time step. Aside from uncertainties in numerical accuracy, this procedure is time demanding as the model resolution is lower. The immediate method to remedy such a problem is to replace the Eulerian advection by the semi-Lagrangian (SL; hereafter) advection with large time step. Pellerin et al. (1995) and Grabowski and Smolarkiewicz (1996) used a SL advection for the cloud model without consideration of mass conservation. Kato (1995) introduced a box-Lagrangian scheme to raindrop advection with mass conservation; however, the box-Lagrangian scheme is a first order for interpolation, which produces diffusive result. There are several high order interpolation schemes, at least higher than first order, that have been used in SL advection with considering mass conservation. These mass conserving SL scheme have been applied to horizontal advection for traces, and efforts to the sedimentation for precipitation have not been given. Despite such advancement in the numerical algorithm, the most cloud schemes in NWP and GCMs still employ the classical Eulerian advection scheme.

This study proposes a SL scheme with higher order interpolation to improve the numerical accuracy and its computation efficiency in computing the sedimentation of falling precipitation drops.

2. Mass conserving tracer equations

In this section, we follow the idea of Juang (2007, 2008) to introduce a one step forward SL scheme with mass conservation and positive definition, and illustrate a necessary modification to take care a leading shock wave to avoid a negative cell advection.

a. Forward semi-Lagrangian advection with mass conservation and positive definition

Since the falling speed depends on the water mass along its path, it is a good approximation to use initial terminal velocity for the advection. Thus we use forward scheme without iteration to determine the path from the departure point, which is the model grid point. The sedimentation of falling precipitation can be written in flux form as

$$\frac{\partial \rho q}{\partial t} = -\frac{\partial \rho q V_T}{\partial z} \qquad (1)$$

where all variables are as usual meteorological use, ρ is density of air, q mixing ratio of precipitation water substance, such as rain, ice, snow or graupel, and V_T is the corresponding terminal velocity. The Eq. (1) can be written in advection form, based on Juang 2008 as

$$\frac{d\rho q\Delta}{dt} = 0 \qquad (2)$$

which is a total derivative or Lagrangian form of flux-form advection of Eq. (1). Since $\rho q\Delta$ represents mass of the precipitation, the Lagrangian form tells us that mass conservation can be obtained in Lagrangian way as

$$(\rho q\Delta)_A = (\rho q\Delta)_D \qquad (3)$$

where sub A and sub D are arrival and departure locations following the terminal velocity in a given time step, and Δ can be replaced by Δz as cell thickness in vertical. Since we use model grid point as departure location, the arrival location has to be interpolated back to model grid point to complete one time step. As long as the interpolation is monotonic, positive definition, we have a forward SL advection for sedimentation of precipitation with mass conservation and positive definition.

Assuming that we obtain terminal velocity at interfaces of model layer at k from nearby terminal velocity, V_T , which is function of sedimentation of precipitation, at model layers by following

$$\hat{w}_k = \sum_{i=k-n}^{k+n-1} \alpha_i V_{T_i} \tag{4}$$

where α is coefficient for interpolation or averaging terminal velocities with order depending on the selection of n and α. After a given time step, the points of interfaces move to arrivals points, $\hat{z}_{A,k}$ and $\hat{z}_{A,k+1}$ by

$$\hat{z}_{A,k+1} = \hat{z}_{D,k+1} - \hat{w}_{D,k+1}\Delta t$$
$$\hat{z}_{A,k} = \hat{z}_{D,k} - \hat{w}_{D,k}\Delta t \tag{5}$$

Thus, we can have discretization of Eq. (3) from departure time n to arrival time n+1 as

$$(\rho q)_{A,k}^{n+1}(\hat{z}_{A,k+1} - \hat{z}_{A,k}) = (\rho q)_{D,k}^{n}(\hat{z}_{D,k+1} - \hat{z}_{D,k}) \tag{6}$$

Since each of the interface for a given model layer is the interface of the neighbor model layers, the mass conservation in each model layer in this Lagrangian advection indicates that total mass is conserved during advection.

b. Modification to avoid negative cell advection

Though mass conservation is discretized in Eq. (6), it does not mean that arrival mass has to be positive. Since departure mass is positive and departure cell is positive as the model layer, the right hand side of Eq. (6) is always positive. However, it is possible that difference of winds at a given cell edges produces negative cell depth at arrival time, then the arrival mass has to be negative to satisfy Eq. (6). To avoid negative mass at arrival time, we have to avoid negative cell depth at arrival time.

To avoid negative arrival mass due to negative arrival cell depth, we obtain following condition

$$(\hat{w}_{D,k+1} - \hat{w}_{D,k})\frac{\Delta t}{\Delta_D} < 1 \tag{7}$$

which was called deformation CFL condition (DeCFL), as mentioned in Xiao et al. (2003). To avoid the violation of Eq. (7), we alter the value of falling speed at the bottom cell edge by scanning from model top to surface as

$$\hat{w}_{D,k} = \hat{w}_{D,k+1} - c\frac{\Delta_D}{\Delta t} \tag{8}$$

where c has to be less than 1, and it is a tunable parameter by experiences. For practical sense, we use 0.05 in our cases, which is the same as 1-λ_2, where λ_2 is 0.95, in Xiao et al. (2003) for their modification of falling displacement.

3. Case results

The SL scheme for sedimentation of precipitating drops, described in the previous section, is evaluated in the WSM3 scheme, against the current method of the Eulerian advection with sub-time steps. Note that, for simplicity and practical sense, forward option without iteration as described in section 2.a and 2.b are used in this section.

We will follow the experimental setup of Xiao et al. (2003) to use their two theoretical cases to examine proposed SL scheme as described in the previous section. The initial distribution of the rainwater is defined (follow Kato 1995) in Xiao et al. (2003) as

$$(\rho q)_k = \begin{cases} 1.2\cos\left[\frac{\pi}{2}\frac{(z_c - z_k)}{D}\right] & for\ |z_c - z_k| \le D \\ 0 & otherwise \end{cases} \tag{9}$$

where ρq is in g/m^3, z_c is 1.0x10^4 m, and D is in m. The computational domain is $0 \le z \le 14000$ m. An equally spaced grid with Δz = 70 m is used for all experiments in this section. Two conditions of terminal velocity are examined, one is a constant terminal velocity as 5 m/s, another one is a function of ρq as

$$V_T = 12.1115(\rho q)^{0.125} \tag{10}$$

which is used in Xiao et al. (2003).

Figure 1 demonstrates that the new method of SL scheme reasonably reproduces the results as shown in Xiao et al. (2003) (in their Fig. 2). It indicates that the higher the order of the interpolation method is, PCM, PLM and PPM, in that order, the less the diffusion occurs as compared to analytical solution. Note that, our PPM with the maxima of 0.9 g is very close to the result from their cubic method with the maxima at 0.8 g, but with higher accuracy. It indicates that our scheme has less diffusive than theirs.

Figure 2 shows the results from the second case with terminal velocity as a function of water density with correction of bottom edge terminal velocity due to DeCFL as mentioned in section 2. with c=0.05. It shows that it is possible for this scheme to have reasonable results even up to 120 s as time step, much larger time step than Xiao et al. (2003). Figure 5 shows the results of comparison between no iteration and iteration with bottom edge terminal velocity correction. The left panels are without iteration, right panels are from two iterations. Then top panel is from PLM and bottom panel is from PPM. No matter which method is used, it indicates that iteration could reduce the unstable noise as shown with time step of 120 s. Furthermore, we put this scheme into WSM6 cloud scheme (Hong and Lim 2006).

4. Discussion

Our theoretical results demonstrate its comparable performance against a conventional Euelrian approach and other SL methods. The third order PPM methods outperforms the PCM and PLM methods, based on the example with constant terminal velocity, even more accurate than Xiao et al. (2003) results in their highest order scheme. The deformation at the leading edge of maximal falling precipitation in the test case shows DeCFL condition to restrict time step within 30 s for our proposed SL scheme. The modification to the falling velocity at the bottom edge of any given cell from model top to model bottom avoid this DeCFL condition, which allows us to large time step up to 120s without a significant altering of the precipitation profile.

Considering both the accuracy and efficiency of the proposed SL scheme within the cloud scheme testbed, the PLM method is a good choice. It is also found that the mean terminal velocity with iteration may not be necessary if stability is not concerned. Thus, the terminal velocity at departure point without iteration is accurate enough for falling precipitation in practice.

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References

Grabowski, W. W., and P. K. Smolarkiewicz, 1996: Two-time-level semi-Lagrangian modeling of precipitation clouds. *Mon. Wea. Rev.*, **124**, 487-497.

Hong, S.Y., and J.-O. J. Lim, 2006: The WRF Single-Moment 6-Class Microphysics Scheme (WSM6). J. Korean Meteor. Soc., 42, 129-151.

Juang, H.-M. H., 2007: Semi-Lagrangian advection without iteration. *Proceedings Conference on Weather Analysis and Forecasting, May 15-17, 2007, Acer Aspire Park, Longtan, Taoyan, Taiwan*, p.277.

________, 2008: Mass conserving and positive semi-Lagrangian tracer advection in NCEP GFS. *Proceedings Conference on Weather Analysis and Forecasting, September 9-11, 2008, Central Weather Bureau, Taipei, Taiwan*, 225-227.

Kato, T., 1995: A box-Lagrangian rain-drop scheme. *J. Meteor. Soc. Japan*, **73**, 241-245.

Pellerin, P., R. Laprise, and I. Zawadzki, 1995: The performance of a semi-Lagrangian transport scheme for the advection-condensation problem. *Mon. Wea. Rev.*, **123**, 3318-3330.

Xiao, F., T. Okazaki, M. Satoh, 2003: An accurate semi-Lagrangian scheme for raindrop sedimentation. *Mon. Wea. Rev.*, **131**, 974-983.

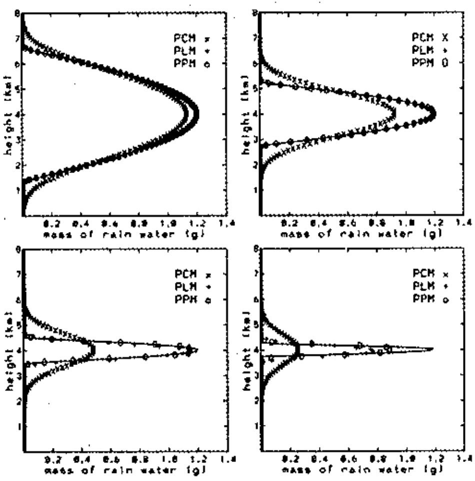

Fig. 1 The theoretical test with constant terminal velocity with different width of rainfall patterns after 20 min integration. Solid curve is from analytical solution in each case, experimental results from piecewise constant method (PCM), piecewise linear method (PLM), and piecewise parabolic method (PPM) are indicated by x, +, and o, respectively.

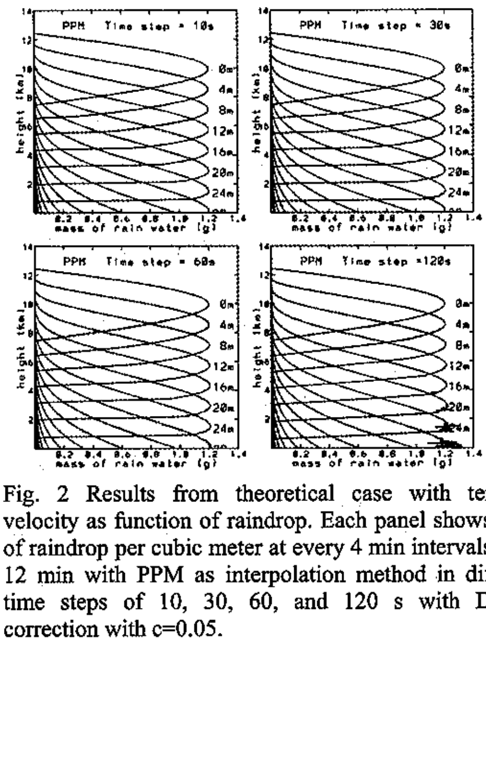

Fig. 2 Results from theoretical case with terminal velocity as function of raindrop. Each panel shows mass of raindrop per cubic meter at every 4 min intervals up to 12 min with PPM as interpolation method in different time steps of 10, 30, 60, and 120 s with DeCFL correction with c=0.05.