MASS CONSERVING AND POSITIVE SEMI-LAGRANGIAN TRACER ADVECTION IN NCEP GFS

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1. Introduction

This extended abstract is to extend the extended abstract of Juang 2007b of CWB (Central Weather Bureau) annual meeting in last year. After we implemented the generalized vertical hybrid coordinates into NCEP GFS (see Juang 2005, 2006), we found some cases during our parallel test for sigma-theta coordinate the model thickness can be negative and it ruined the model integration. Instead of arbitrary fix it, we found mass conserving positive advection may be able to avoid it. To have mass conserving positive advection, we may have to use semi-Lagragian to save time in advection. Instead of using existed mass conserving positive advection in the literature, such as SJ Lin's work (Lin and Rood 1997) and others, the non-iteration semi-Lagragian advection presented in the previous meeting is extended to have mass conservation and positive advection.

In this extended abstract, we will provided basis of the mass conservation from the differential equation for generalized vertical coordinates in section 2, then illustrates how the mass conservation can be done from flux form equation by semi-Lagrangian advection. Also, we briefly explain the condition to have mass conservation in section 3. Some idealized cases are tested and described, but not shown here. The detail of implementation into NCEP GFS is not shown, but a case result demonstrates mass conserving and positive advection in section 4. Some concerns and future works is in section 5.

2. Mass conserving tracer equations

For tracer equation without source and sink, it is simply written as

$$\frac{\partial q}{\partial t} = -m^2 u^* \frac{\partial q}{a\partial\lambda} - m^2 v^* \frac{\partial q}{a\partial\varphi} - \dot{\zeta}\frac{\partial q}{\partial\zeta}$$

where q is specific value of any given tracer, and the continuity or density equation in generalized vertical coordinates as shown in Juang 2005 can be written as

$$\frac{\partial(\partial p/\partial\zeta)}{\partial t} = -m^2\left(\frac{\partial u^*(\partial p/\partial\zeta)}{a\partial\lambda} + \frac{\partial v^*(\partial p/\partial\zeta)}{a\partial\varphi}\right) - \frac{\partial\dot{\zeta}(\partial p/\partial\zeta)}{\partial\zeta}$$

Combining above two equations, we can have flux form of tracer density equation as

$$\frac{\partial(\partial p/\partial\zeta)q}{\partial t} = -m^2\left(\frac{\partial u^*(\partial p/\partial\zeta)q}{a\partial\lambda} + \frac{\partial v^*(\partial p/\partial\zeta)q}{a\partial\varphi}\right) - \frac{\partial\dot{\zeta}(\partial p/\partial\zeta)q}{\partial\zeta}$$

Integral globally the above equation, we obtain

$$\oint\!\!\iiint \frac{\partial(\partial p/\partial\zeta)q}{\partial t} a^2 \frac{1}{m^2} d\lambda d\varphi d\zeta = 0$$

It shows global mass conservation of any giving tracer. To maintain this conservation, any descretization for numerical model has to provide the same characteristic of this mass conservation. In the next section, we will show how to have a descretization to maintain this characteristic.

3. Mass conserving positive semi-Lagrangian advection

We will follow the method described in Juang 2007b to do semi-Lagrangian advection without iteration to find the mid-point wind. However, it started from advection form to do semi-Lagrangian, and spatial splitting. Since mass conserving can be easily obtained with flux form as the previous equation, let's start from the previous flux form equation but consider advection form of some alternative, thus it can be written in one direction for example as

$$\frac{\partial A}{\partial t} = -\frac{\partial uA}{\partial x} = -u\frac{\partial A}{\partial x} - A\frac{\partial u}{\partial x}$$

where, for example,

$$A = \frac{\partial p}{\partial\zeta}q$$

$$x = \lambda$$

And the divergence term can be written as

$$\frac{\partial u}{\partial x} = \frac{1}{\Delta x}\frac{d\Delta x}{dt}$$

Then the flux form tracer density equation can be written in advection form with the consideration of mass conversation as following

$$\frac{dA\Delta x}{dt} = 0$$

so the final form for semi-Lagrangian equation become

$$(A\Delta x)^{n+1}_{arrival-point} = (A\Delta x)^{n-1}_{departure-point}$$

which tell us mass (tracer density A times volune) is conserving in semi-Lagrangian advection. As long as we provide conditions during interpolation among following,

$$\sum_{global-sum}(A\Delta x)_{departure-point} = \sum_{global-sum}(A\Delta x)_{regular-point}$$

$$\sum_{global-sum}(A\Delta x)_{regular-point} = \sum_{global-sum}(A\Delta x)_{arrival-point}$$

then the total global tracer mass should be conserved. The interpolation method to have mass conservation, we can adopt monotonic piece-wise parabolic method (PPM) to construct conserving and positive interpolation with up to 3^{rd} order accuracy. The edge of any given cell at departure, regular, and arrival condition is determined by the wind speed at the edge of the middle time step from regular model cell. Details can be found in presentation or future publication. And we name this method as nislfv (non-iteration semi-Lagrangian finite volume) advection.

4. Case results

The case with global tracer, which was tested in the previous non-iteration semi-Lagrangian, is tested here with this mass conserving and positive advection. The another case is the same test but with a random flow, which has isochronal character in certain preset time period, the flow comes back to its original location. The mass conserving error in these two cases are about the same order of truncation error. It is 10 to -15 in double precision computation. We can say it is conserving up to truncation error.

After testing with idealized cases, we implemented the method into NCEP GFS only for tracers, which includes specific humidity, cloud water, and ozone. We provide initial condition of tracers in grid-point space without negative value. All computations related tracer in spectral space are skipped. We expect current method give us all positive tracer at all time, and we can compare it with control NCEP GFS, which has spectral computation on tracers. Figure 1 shows the cloud water after 6 hr integration at model layer number 35 from control run (NCEP GFS with spectral computation for tracers). The shaded areas with green color are negative values. The wiggling of the green color areas indicates the resulting from spectral computation.

Figure 2 shows the same plot as in Fig. 1, except it is from the current method without spectral computation for tracers. There is no green shaded area, and the patterns of the cloud water are very similar to those shown in Fig. 1. Figure 3 and 4 are the same as Figs. 1 and 2, respectively, except from 72 hr integration at model layer number 5. Since the tracers have influence to the thermodynamics nonlinearly through enthalpy (Juang 2007a), the results from 72 hr show more difference between control and experiment runs, however, there is no negative values from the experimental result.

5. Discussion

The non-iteration semi-Lagrangian finite volume tracer advection is presented. The flux form advection equation is used for mass conservation, however, the Lagrangian form of finite volume or grid cell for divergence changes results a semi-Lagrangian equation in advection form for flux form conserving equation. The PPM with monotonicity provides third order accuracy positive advection.

Several tests with this mass conserving positive tracer advection with non-iterative semi-Lagrangian method show reasonable performance. The specific case with isochronal flow demonstrates accepted results through very random flow, instead of linear flow. The test in NCEP GFS showed no negative tracer and reasonable tracer advection.

Since there is time consuming by PPM, semi-Lagragina of entire model should be implemented, not only for tracers. The descretization with non-iterative semi-Lagrangian finite volume advection are derived with concerning all conservations, and it is ready to implement into the NCEP GFS, which will be the extended work from current results for future presentation.

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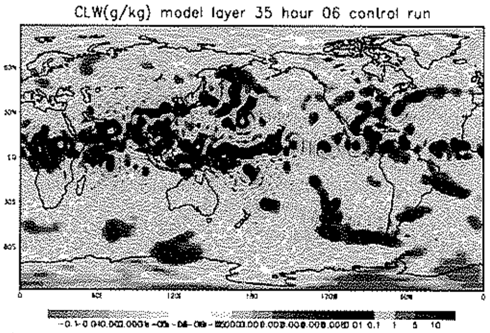

Fig. 1 Cloud water in g/kg at model layer number 35 after 6 hr integration by spectral computation in NCEP GFS.

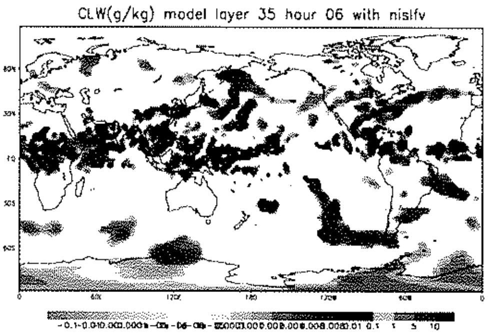

Fig. 2 Cloud water in g/kg at model layer number 35 after 6 hr integration by non-iteration semi-Lagrangian mass-conserving positive advection implemented in NCEP GFS.

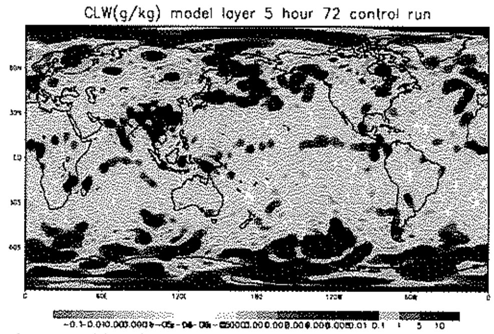

Fig. 3 Cloud water in g/kg at model layer number 5 after 72 hr integration by spectral computation in NCEP GFS.

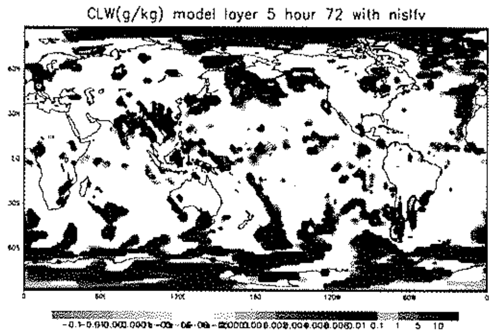

Fig. 4 Cloud water in g/kg at model layer number 5 after 72 hr integration by non-iteration semi-Lagrangian mass-conserving positive advection implemented in NCEP GFS.