A Dynamical Study for the Boundary Layer of Cold Front

Ming Zhao

Department of Atmospheric Sciences, Nanjing University, Nanjing,210093,China

1. Introduction

The current research on the dynamics of front mainly is about the theory of frontogenesis, the research about the dynamical characteristics of the front in boundary layer is not sufficient. Gutman (1976) studied the stream field for the front in the boundary layer by using classical motion equation for the boundary layer. The inertial force in the equation of boundary layer was not included in previous studies. In this paper, the equation of PBL with geostrophic momentum approximation (GMA) (Wu and Blumen, 1982) is used to study the motion in the boundary layer of a cold front, and in general, this research may also be extended to the case that the geostrophic wind varies with height, then we can obtain the effects of the temporal and spatial variations of geostrophic wind on the dynamical characteristics of the front.

2. The boundary layer equation with GMA for front

As Gutman (1976), assume that the front which is along y-axis is travelling along x-axis with a constant speed *c*, the form of the front does not change in the travelling process. For 2 dimensional problem the boundary layer equation with GMA when front exists is as follows:

$$K\frac{f^2u_i}{fz^2}+a_1u_i+b_1v_i=c_1 \qquad (1)$$

$$K\frac{f^2v_i}{fz^2}+a_2u_i+b_2v_i=c_2 \qquad (2)$$

where

$$a_1=-fu_g/fx,\ b_1=-fu_g/fy+f,$$

$$c_1=fv_g+fu_g/ft+\mu fh/fx\delta_{i1}$$

$$a_2=-f-fv_g/fx,\ b_2=-fv_g/fy,\ c_2=-fu_g+fv_g/ft \qquad (3)$$

i =1,2 represent the air below and above front surface respectively, h is the height of the front surface, $\mu=-g\Delta\theta/\Theta$, $\Delta\theta$ is the difference of potential temperature between warm and cold air, the eddy exchange coefficient K is assumed as a constant.For two dimensional problem, $a_1=b_2=0, b_1=f$ ·

At the upper boundary, the wind speed is taken as the wind speed in free atmosphere with GMA:

$$u_2=u_T=(c_3b_2-b_1c_2)/D^4=-b_1c_2/D^4,$$

$$v_2=v_T=(a_1c_2-c_3a_2)/D^4=-c_3a_2/D^4 \text{ where } z\_\_ \qquad (4)$$

where c_3 is c_1 for i=2,

$$D^4=a_1b_2-a_2b_1=-a_2b_1$$

at the lower boundary :

$$u_1=v_1=0 \quad \text{where } z=0 \qquad (5)$$

the connecting condition is:

$$u_1=u_2, v_1=v_2 \quad \text{at } z=h \qquad (6)$$

The solution of Eqs.(1),(2) is:

$$u_1=u_T'(1-e^{-\varsigma}\cos\varsigma)-c_1D^{-2}e^{-\varsigma}\sin\varsigma \qquad (7)$$

$$v_1=v_T'(1-e^{-\varsigma}\cos\varsigma)-c_1D^{-2}e^{-\varsigma}\sin\varsigma \qquad (8)$$

for $z\le h$ and

$$u_2=u_1(\varsigma)+\frac{\mu fh/fx}{D^2}e^{-(\varsigma-\eta)}\sin(\varsigma-\eta) \qquad (9)$$

$$v_2=v_1(\varsigma)+\frac{a_2\mu fh/fx}{D^4}[1-e^{-(\varsigma-\eta)}\cos(\varsigma-\eta)] \qquad (10)$$

for $z>h$,where $\varsigma=\beta z, \eta=\beta h, \beta=D/\sqrt{2K}$ and

$$u_T'=-b_1c_2/D^4, v_T'=-c_1a_2/D^4$$

It is easy to prove that (7)-(10) satisfy all boundary and connecting conditions.

3.The slope and stream field for a cold front

Similar to Gutman (1976), based on (7), we can obtain the slope:

$$\frac{f\eta}{f\xi}=-\frac{1+\alpha-2(1-C)\eta-e^{-\eta}[(1+\alpha)\cos\eta+(\alpha-1)\sin\eta]}{D^{-2}f[1-e^{-\eta}(\cos\eta+\sin\eta)]} \qquad (11)$$

where

$$\alpha=(fv_g+fu_g/ft)D^{-2}/u_T \qquad (12)$$

and $C=c/u_T, \xi=f\beta xu_T/\mu$.

For cold front, $f\eta/f\xi<0$, mathematical analysis of (11) shows that when η ♦ 0 ,$f\eta/f\xi$ ♦ × , i.e., the front surface is almost perpendicular to the ground where the front surface is near it which is supported by recent observations (Shapiro,1984,Miao1994).Because

$$D^4=(f+fv_g/fx)f$$

then D is greater for positive geostrophic vorticity, from (11), the slope increases, the reverse is true for negative geostrophic vorticity. Numerically integrating Eq.(11), we may obtain the profile of front surface, it also shows that the slope increases towards the ground.

The parameters C_1 (or C_3) and C_2 contain the tendency of geostrophic wind speed, so that the tendency will affect the slope and stream field. $fu_g/ft>0$ and $fv_g/ft>0$ will increas the slope.

Defining stream function in the coordinate system travelling with the front,we can find the stream line in the coordinate system travelling with the front. The stream field for the case C<1, for example,C=0.8 shows that there exist downslide motion and clockwise circulation below the front surface, but upslide motion exists in a layer above the front surface. For the cases with $C>1$, for example $C=1.5$, because the speed of front is greater than wind along x-axis, the domain of upslide motion is greater, all the motion above the front surface has negative x – component.Although the stream field shows different characteristics for different parameters, however, there are common characteristics, i.e., there exist downslide motion and clockwise circulation below the front surface and upslide motion above it.

4.The case for varying geostrophic wind

Now consider the case that large-scale geostrophic wind varies with height and the temperature difference across the front is not affected. For simplicity, we assume that the geostrophic wind components are the linear function of height, the derivatives of the geostrophic wind in (3) are taken as their averages for different heights. Similarly, we can find the solution of Eqs.(1),(2), consequently, find the slope of the front surface and the stream field. The results show that only the vertical variation of the geostrophic wind in x-axis can affect the slope, the decrease of u_g with height can increase the slope. The stream line below the front surface is similar to that in section 3 for the case that the geostrophic wind component in x-axis increases with height, but for the reverse case, there exists upslide motion below the front surface far from the location of the ground front. Above the front surface, when the geostrophic wind component in x-axis increases with height, the upslide motion decreases, the reverse is true, all of these can be explained physically.

References

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