Derivation of Thermodynamic Fields in a Three Dimensional Space Using Doppler Wind Observations

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ABSTRACT

This paper presents a newly designed thermodynamic retrieval algorithm whereby one can deduce the potential temperature and pressure gradient fields in a three dimensional space using only Doppler radar wind measurements. In order to achieve this purpose, all three equations of motion are implemented in a single cost function. Then, given the detailed wind information, these equations are solved simultaneously in a least square sense. The products of the approximated solutions are the three dimensional potential temperature fluctuations, and the pressure gradients along any direction. Compared to the traditional method proposed by Gal-Chen, the advantage of this scheme is that the vertical structure of the thermodynamic variables can be determined without requiring a point of independent observation of the pressure and temperature for each layer. It is believed that this algorithm can be of particularly useful in many Doppler weather radar applications.

1. Introduction:

Although Doppler radar can provide information about wind with a fine spatial and temporal resolution, the structure of thermodynamic variables, such as temperature and pressure, cannot be directly detected. The pioneer work by Gal-Chen (1978, hereafter GC) demonstrated that given the detailed Doppler wind observations, and their time derivatives through consecutive volume scans, one could extract the information of the pressure and temperature perturbations. This powerful method turned out to be particularly popular, and was later widely adopted by many researchers to deal with various weather phenomena. Despite the encouraging results, it should be emphasized that GC only retrieves the deviations (from the horizontal mean) of the pressure and temperature perturbations (with respect to a base state) for each horizontal plane. That is, only $\theta' - \langle\theta'\rangle$ and $p' - \langle p'\rangle$ are solved uniquely at each horizontal layer, where $\langle\ \rangle$ stands for a spatial average over that layer. In order to specify the values of $\langle\theta'\rangle$ and $\langle p'\rangle$, and determine the absolute pressure and temperature fluctuations θ' and p', at least one point of independent field measurement of the temperature and pressure for each altitude must be available. Although a sounding may provide such information, unfortunately, considering the scenario of daily operations, difficulties persist. The consequence of this drawback is that, whenever an interpretation of the ***vertical structure*** of the thermodynamic fields is required, one must proceed with caution.

A new approach was proposed by Roux (1985, 1988), whereby the pressure and temperature perturbations can be solved uniquely up to a single constant. To deduce this unknown constant requires only one independent pressure and temperature observation at a single point somewhere in the retrieval domain. Roux and Sun (1990) modified this method by taking into account a simplified form of the thermodynamic equation everywhere in the domain, so that the temperature gradient was provided both horizontally and vertically. On the other hand, with the rapid development of the adjoint method, Sun and Crook (1996) demonstrated the advantages of using the so-called 4D-Var technique over the traditional GC scheme for retrieving the thermodynamic field. In contrast to the previous techniques, an important feature shown by the 4D-Var formulation is that the absolute temperature perturbations can be inferred without the auxiliary of any extra, independent measurements.

In this research, a different approach is suggested. The basic methodology involves implementing the horizontal and vertical equations of motion into a single cost function. Through the method of variational analysis, a set of optimally determined potential temperature fluctuations and pressure perturbation gradients that minimizes this cost function, can be deduced simultaneously.

The proposed method and a simple experiment to show its usefulness are presented in this paper.

2. Methodology:

The basic momentum equations may be written as follows:

$$\frac{1}{\theta_0}\left[\frac{Du}{Dt} - fv + turb(u)\right] = -\frac{\partial \pi'}{\partial x} \equiv -F \quad (1)$$

$$\frac{1}{\theta_0}\left[\frac{Dv}{Dt} + fu + turb(v)\right] = -\frac{\partial \pi'}{\partial y} \equiv -G \quad (2)$$

$$\frac{1}{\theta_0}\left[\frac{Dw}{Dt} + turb(w)\right] = -\frac{\partial \pi'}{\partial z} + g\frac{\theta'}{\theta_0^2} \equiv -H \quad (3)$$

where π is called the Exner function, or a normalized form of pressure, and *turb( )* denotes a sub-grid scale turbulence parameterization operator. Note that the subscript "o" represents a horizontally homogeneous basic state, from which the non-hydrostatic perturbations are expressed by variables with a single prime. Since F, G and H are functions of the wind only, their values can be estimated when the three-dimensional air motion is known through either the multiple-Doppler radar synthesis procedure, or a single Doppler wind retrieval.

In addition to the momentum equations, a simplified thermodynamic equation is employed to provide the temperature gradient information. Neglecting the time evolution, diffusion, and source-sink terms, this equation is expressed as:

$$u\frac{\partial \theta'}{\partial x} + v\frac{\partial \theta'}{\partial y} + w\frac{\partial \theta'}{\partial z} + w\frac{d\theta_0}{dz} = 0 \quad (4)$$

Adopting Eqs. (1) ~ (4), one formulates a cost function as in the following:

$$J = \int \left(\alpha_1 P_1^2 + \alpha_2 P_2^2 + \alpha_3 P_3^2 + \alpha_4 P_4^2 + \alpha_5 P_5^2 + \alpha_6 P_6^2 + \alpha_7 P_7^2\right) d\Omega \quad (5)$$

$$d\Omega = dxdydz$$

$$P_1 = \left(\frac{\partial \pi'}{\partial x} - F\right)$$

$$P_2 = \left(\frac{\partial \pi'}{\partial y} - G\right)$$

$$P_3 = \left(\frac{\partial \pi'}{\partial z} - g\frac{\theta'}{\theta_0^2} - H\right)$$

$$P_4 = \left(u\frac{\partial \theta'}{\partial x} + v\frac{\partial \theta'}{\partial y} + w\frac{\partial \theta'}{\partial z} + w\frac{d\theta_0}{dz}\right)$$

$$P_5 = \left(\frac{\partial\theta'}{\partial z}\right)$$

$$P_6 = \frac{\partial^2\pi'}{\partial x^2} + \frac{\partial^2\pi'}{\partial y^2} + \frac{\partial^2\pi'}{\partial z^2}$$

$$P_7 = \frac{\partial^2\theta'}{\partial x^2} + \frac{\partial^2\theta'}{\partial y^2} + \frac{\partial^2\theta'}{\partial z^2}$$

In Eq. (5), the first two terms (P_1^2, P_2^2) measure the distance between the retrieved horizontal pressure gradients and the calculated F and G (from the wind observations), respectively. P_3^2 denotes the difference in the retrieved vertical pressure gradient, the buoyancy, and the quantity H. P_4^2 represents the thermodynamic equation, which offers a constraint throughout the potential temperature field's 3-D spatial distributions. Constraint P_5^2 can effectively eliminate the "spurious" thermal instability, while the "true" thermally unstable region is kept intact. Finally, constraints P_6^2 - P_7^2 are added as low-pass filters to suppress noise. Through the process of variational adjustment, the cost function shown by Eq. (5) can be minimized. The solution is a set of optimally determined pressure and potential temperature perturbations that satisfy these weak constraints simultaneously in a least square sense.

A close examination of the Eq. (5) reveals that the potential temperature appears in its *absolute* form, but the pressure is always expressed by first order derivatives. Consequently, when Eq. (5) is minimized, the results will provide the structure of the buoyancy field in a three-dimensional space, as well as the pressure fluctuations gradients, not only for each horizontal plane, but also in the vertical direction. In addition, unlike the techniques suggested by Roux (1988), this method does not require higher order spatial derivatives of the momentum equations, which we believe is a procedure to amplify the impacts of the observational errors when real Doppler radar wind data are utilized.

3. Results and Summary:

In this paper the data sets utilized for validation come from the numerical simulation of a collapsing cold pool, a frequently observed meso-scale phenomenon. The model domain covers a three-dimensional space with a volume of approximately $55 \times 55 \times 8$ km^3. The horizontal and vertical resolutions are 1000m and 150m respectively. Figure 1 shows the model generated potential temperature perturbations θ' on a x-z plane, crossing through the center of the cold pool. This picture is selected as the "true" solution. Fig. 2 illustrates the retrieved buoyancy field using the method of GC. Note that without the knowledge of $<\theta'>$, this is actually the vertical structure of θ' - $<\theta'>$, instead of θ' itself. One immediately recognizes that the differences between these two pictures are significant. Finally, Fig. 3 displays the retrieved temperature distributions using the algorithm proposed in this paper. The consistency with the true solution is obvious.

This experiment demonstrates that the method presented in this paper is a feasible tool to extract the thermodynamic information in a three dimensional space using only Doppler radar observations. We believe this algorithm can be of particularly useful in many Doppler weather radar applications.

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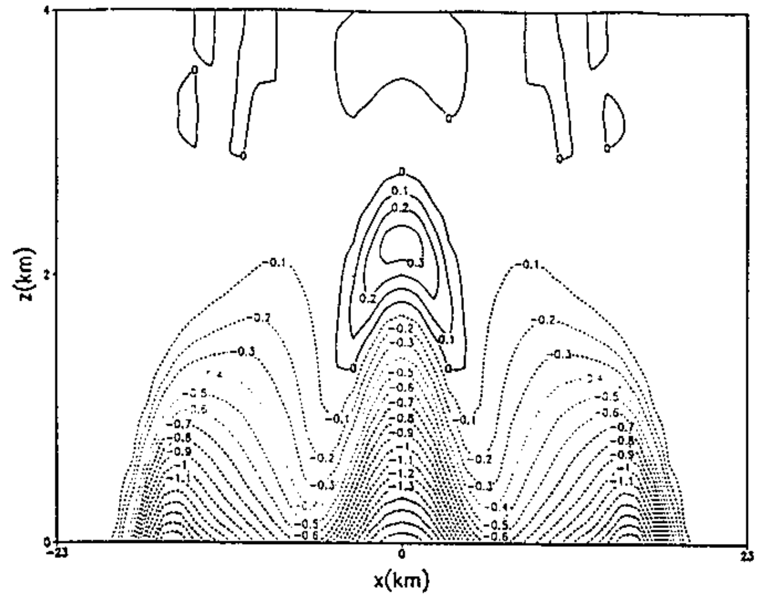

Fig.1 The "true" potential temperature perturbations in a x-z cross section, generated by a numerical model. The contour interval is 0.1°

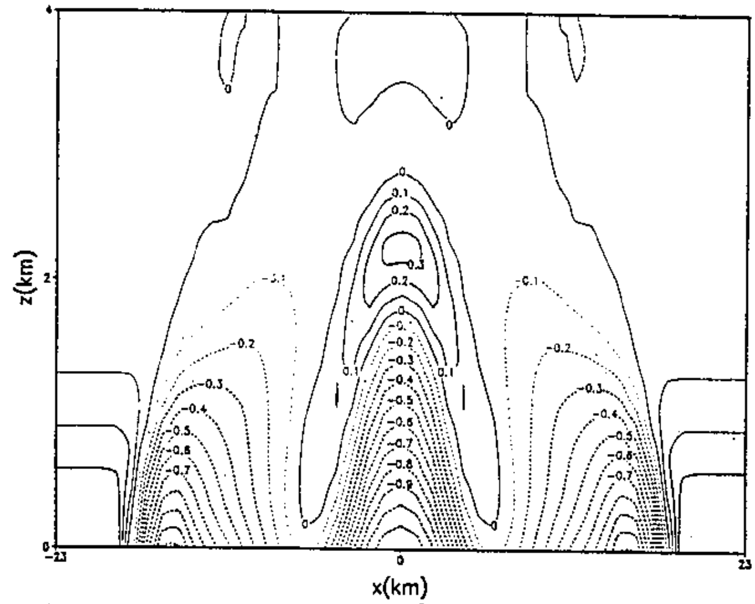

Fig.2 The retrieved potential temperature perturbations using the method of GC.

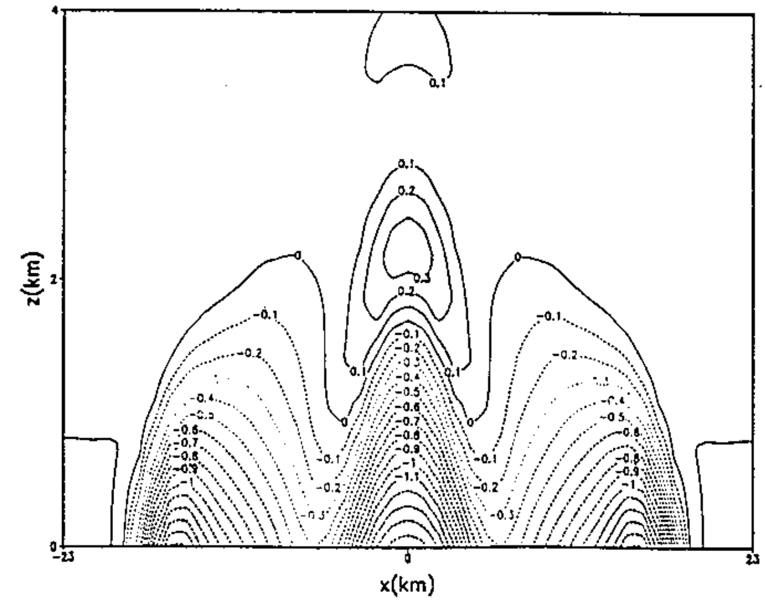

Fig.3 The retrieved potential temperature perturbations using the method proposed in this paper.