Single Radar Wind Retrieval - The Method of Using a Lagrangian Coordinate

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ABSTRACT

A moving frame of reference technique is proposed so that the unobserved cross-beam wind components can be retrieved, with high resolution, from data measured by either single Doppler or conventional Radar. In this algorithm the reflectivity fields detected by consecutive radar scans are used to find a Lagrangian coordinate for which the reflectivity measurements are as stationary as possible. After interpolating all the observational data onto this optimal moving frame, one can formulate a cost function with the following constraints: (1) Conservation of reflectivity; (2) A geometric relationship between the radial velocity *Vr* and its Cartesian components *u*, *v*, *w* ; (3) Incompressibility; (4) Weak vertical vorticity. By minimizing this cost function, a complete three dimensional wind field can be constructed. Using simulated data to test this method, it is found that: (1) The retrieval errors can be reduced on a moving frame, and this algorithm performs well in recovering the cross-beam wind component ; (2) When only reflectivity data are available, this method is still capable of catching the principal features embedded in the true wind field ; (3) The computation is economical. Overall, this method is highly applicable for the purposes of daily operations.

I. Introduction:

The major limitation of Doppler radar is that the observed data are incomplete. Only the radial component (V_r) of a three-dimensional wind vector can be detected. The unobserved cross-beam components, that is, the azimuthal (V_ϕ) and elevational (V_θ) velocities, need to be retrieved by other means.

In recent years, several revolutionary techniques have been developed to solve the single-Doppler wind retrieval problem. Among them, the assumption of Lagrangian conservation for reflectivity (η):

$$\frac{\partial\eta}{\partial t}+u\frac{\partial\eta}{\partial x}+v\frac{\partial\eta}{\partial y}+w\frac{\partial\eta}{\partial z}=0 \qquad (1)$$

is frequently used (e.g. Tuttle and Foote, 1990 ; Sun et. al. 1991 ; Qiu and Xu, 1992 ; Xu et. al., 1994 ; Shapiro et. al. 1995, etc.). Here *u*, *v*, and *w* denote the wind velocities along the *x*, *y*, and *z* coordinates respectively, and *t* stands for the time. However, the conservation is valid only in some special cases. In general situations, (1) is merely an approximation.

Due to the discreteness of radar measurements, any estimates of local temporal derivatives from radar data obtained by finite differencing two observations from consecutive volume scans may lead to large errors. Gal-Chen (1982) suggested that this error can be significantly reduced by finding a set of optimal velocities on which the reflectivity field is advected while (1) is least violated. Based on this concept, Zhang and Gal-Chen (1996) developed a moving frame algorithm that retrieved 3-D wind from single radar observations. In this study we propose a modified version of the moving frame technique in order to further improve the quality of the retrieved wind field.

II. Optimal Advection Velocity:

As suggested by Gal-Chen (1982), a set of optimal advection velocities *U*, *V*, *W* can be obtained by minimizing the local temporal derivative of a scalar field (or reflectivity in the radar observation) in a moving frame. This is equivalent to rewriting the above statement on a fixed frame as:

$$J(U,V,W)=\int_\Omega \alpha\left[\frac{\partial\eta}{\partial t}+U\frac{\partial\eta}{\partial x}+V\frac{\partial\eta}{\partial y}+(W+W_t)\frac{\partial\eta}{\partial z}\right]^2 dxdydzdt \qquad (2)$$

W_t represents the terminal velocity of the precipitation particles, and can be estimated empirically from the reflectivity data. Ω covers the whole area where the retrieval is intended, and extends in the time domain from the first to the last radar scans. α is a weighting coefficient to balance the magnitude of each penalty term in (2). Setting the derivatives of *J* with respect to *U*, *V*, *W* at zero, (2) gives analytically the optimal advection wind, and a new reference frame can be formed by:

$$\begin{aligned} x' &= x-U(t-t_0) \\ y' &= y-V(t-t_0) \\ z' &= z \\ t' &= t \end{aligned} \qquad (3)$$

Here t_o is the reference time, and the prime stands for quantities on a moving frame. All the radar data are then redefined in this moving frame.

III. Retrieving Perturbation Velocities:

The true wind field is decomposed into:

$$
\begin{aligned}
u &= U + u' \\
v &= V + v' \\
w &= W + w'
\end{aligned}
\qquad (4)
$$

Here u', v', w' are the unknown perturbation velocities. For a retrieval domain, a cost function is defined by:

$$J = \frac{1}{2} \int_T \int_\Omega (\alpha_1 J_1 + \alpha_2 J_2 + \alpha_3 J_3 + \alpha_4 J_4)\, dxdydzdt \qquad (5)$$

where

$$J_1 = [\frac{\partial \eta'}{\partial t'} + u' \frac{\partial \eta'}{\partial x'} + v' \frac{\partial \eta'}{\partial y'} + (w' + W_t) \frac{\partial \eta'}{\partial z'}]^2$$

$$J_2 = (v'_\gamma - u' \frac{x' - p'_1}{\gamma'} - v' \frac{y' - p'_2}{\gamma'} - w' \frac{z' - p'_3}{\gamma'})^2$$

$$J_3 = (\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} + \frac{\partial w'}{\partial z'})^2$$

$$J_4 = (\frac{\partial v'}{\partial x'} - \frac{\partial u'}{\partial y'})^2$$

p'_1, p'_2, and p'_3 are the Cartesian coordinates of the radar in the moving frame, while r' is the distance between the radar and the grid point. They are defined by:

$$
\begin{aligned}
P_1' &= P_1 - U(t - t_o) \\
P_2' &= P_2 - V(t - t_o) \\
P_3' &= P_3 \\
r' &= \sqrt{\sum_{i=1}^{3} (x_i' - p_i')^2} \\
V_r' &= V_r - U \frac{x' - p'_1}{r'} - V \frac{y' - p'_2}{r'} - W_t \frac{z' - p'_3}{r'}
\end{aligned}
$$

Equation (5) simply states that in the moving frame the conservation of reflectivity, as well as the geometric relationship between the radial velocity (Vr') and its Cartesian components (u', v', w'), should be satisfied as much as possible. J_3 is implemented in (5) to reduce the divergence, while J_4, the weak vertical vorticity condition, is added to serve as a smoothness constraint to prevent ill-conditionness and suppress the noise (Sasaki, 1970; Qiu and Xu, 1996). By minimizing (5), one gets a set of optimal u', v', w' for each grid point.

IV. Retrieval Experiments:

1. Data Sets:

A numerical model is integrated to generate artificial data sets for testing the retrieval schemes. Four data sets, which contain radial velocity and reflectivity, are collected by an imaginary radar located 10 km southwest of the lower left corn of the domain..

2. Results:

The experiments are organized into three groups. Group A is designed to investigate the influence of using a moving frame. Figure 1 shows the modeled flow pattern, which is assumed as the "true" wind field. Since the radar is placed southwest from origin, most of the wind vectors are perpendicular to the radar beam. Figure 2 shows the retrieved wind when the frame is fixed with respect to the ground, and the results can be seen to deviate seriously from the true field. However, when a moving frame is applied, the retrieved field is substantially improved, as illustrated by Figure 3.

In group B, a more sophisticated wind field, displayed in Figure 4, is employed to test this method. The moving frame technique uses different number of data sets to conduct the retrieval, and the statistics are listed in Table 1. The spatial correlation coefficient (SCC) score for a variable A is defined by:

$$SCC(A) = \frac{\sum (A_r - \overline{A_r})(A_t - \overline{A_t})}{\sqrt{\sum (A_r - \overline{A_r})^2 \sum (A_t - \overline{A_t})^2}} \qquad (6)$$

with the subscripts "t" or "r" stand for "true" or "retrieved" field, respectively. The overbar denotes an spatial average. Figure 5 depicts the result with only two volume scans. It is found that the gross picture of the flow pattern is successfully inferred. When more data sets are available, Table 1 illustrates that the quality of the retrieved wind can be significantly improved. Figure 6 shows the graphic result with four scans. The SCC score in this case reaches a very high level (~ 0.9). Note that the resolution of the retrieved field remains the same as the radar data.

Table 1 : Statistical Result from Group B

No. of scans	$\overline{V_\phi}$ (m/s)	SCC (V_ϕ)	rms V_ϕ (m/s)
2	0.82	0.71	2.73
3	1.99	0.86	1.62
4	2.32	0.89	1.42
true	2.58		

Experiments in Group C explore the performance of this method when only reflectivity data are available. Throughout the whole retrieval process, the information about the radial velocities is only required in (5) when calculating J_2, and the weighting coefficients α_3 and α_4. By setting α_2 at zero, one eliminates contributions from the radial velocity constraints. Unfortunately, without the radial wind, the measurements of the divergence and vertical vorticity, which are needed for estimating the weighting coefficients, is impossible. Thus, we select a rough estimate of the divergence, and the weighting coefficients are defined as:

$$\alpha_3 = C/10^{-5} \tag{7}$$

$$\alpha_4 = 0.01\alpha_3 \tag{8}$$

Here, C is a constant.

Assuming only three radar volume scans are conducted. The retrieved wind field with $C=1$ is displayed in Figure 7. It is shown that several major features embedded in the true wind field, that is, the cyclone-type flow pattern in the northwestern part of the domain, and the prevailing southwestern-western-northwestern wind field in the lower portion of the domain, are all successfully reproduced. To examine the sensitivity of C, a series of experiments are conduced, and it is realized that the retrieval results are acceptable as long as C is approximately within the 0.1 to 5.0 range. Therefore, this retrieval algorithm has demonstrated its ability in reconstructing the complete flow features when only reflectivity data observed by single radar are available. In other words, the utility of conventional radar can be further promoted.

V. Summary:

A moving frame technique is proposed by which one can recover the cross-beam wind components from single radar observations. Using simulated data to test this method, it is found that:

1. The errors in the retrieved field can be reduced by applying a Lagrangian frame of reference.
2. When applied to Doppler data, this method successfully reconstructs the complete flow field without sacrificing the radar data resolution.
3. When only reflectivity data are available, and the divergence and vorticity are roughly estimated, this method still works well, at least qualitatively, in retrieving a gross picture of the total wind field. Therefore, the application of this method to conventional radar is feasible.
4. The computation is economical. In this study, each experiment took less than 300 seconds of CPU time on an IBM 580 work station.

Based on the above summaries, it is believed that the method reported in this study is highly applicable to the situations of daily operations.

VI. Acknowledgment:

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VII. Reference:

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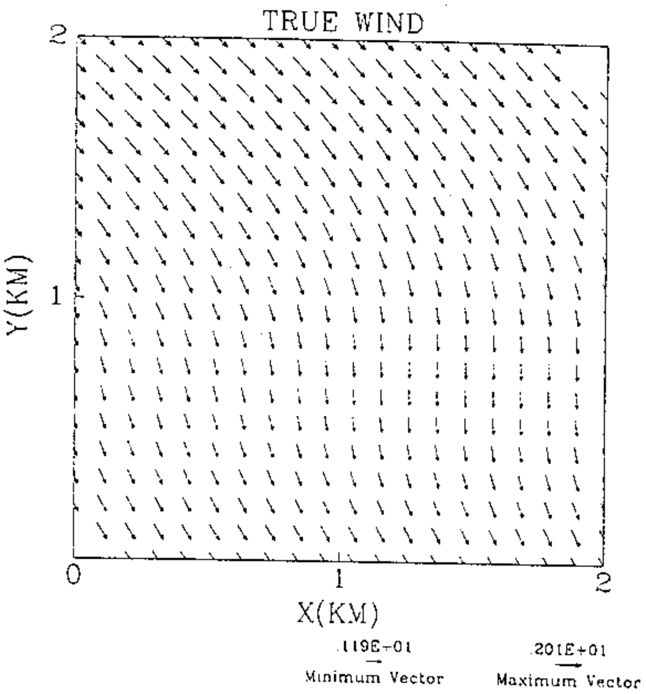

Fig. 1 Modeled "true" wind field for Group A experiments..

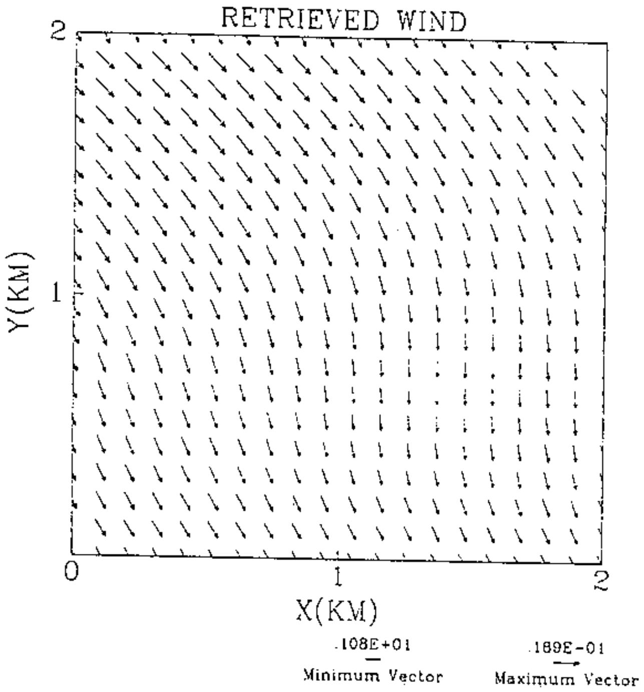

Fig. 3 Same as Fig. 2, but WITH moving frame.

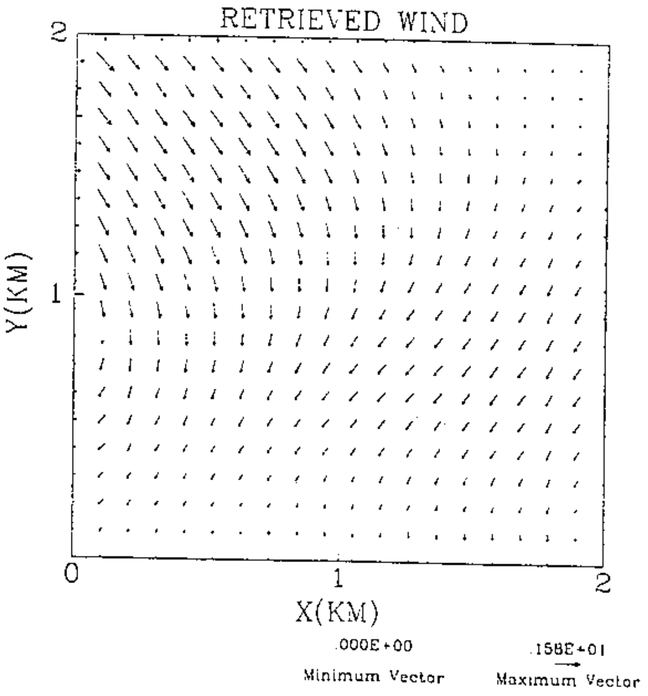

Fig. 2 Retrieved wind WITHOUT moving frame in Group A experiments..

Radar is located southwest of the lower-left corner of the retrieval domain

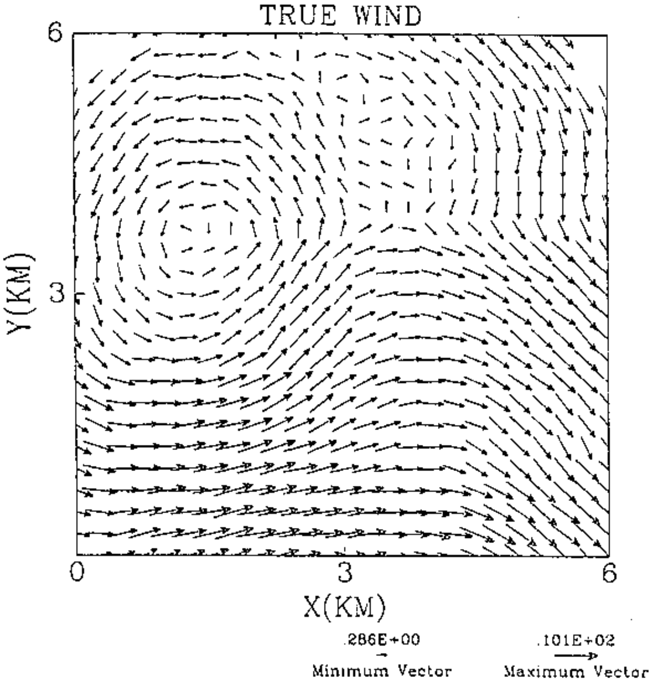

Fig. 4 Modeled "true" wind field for Group B experiments.

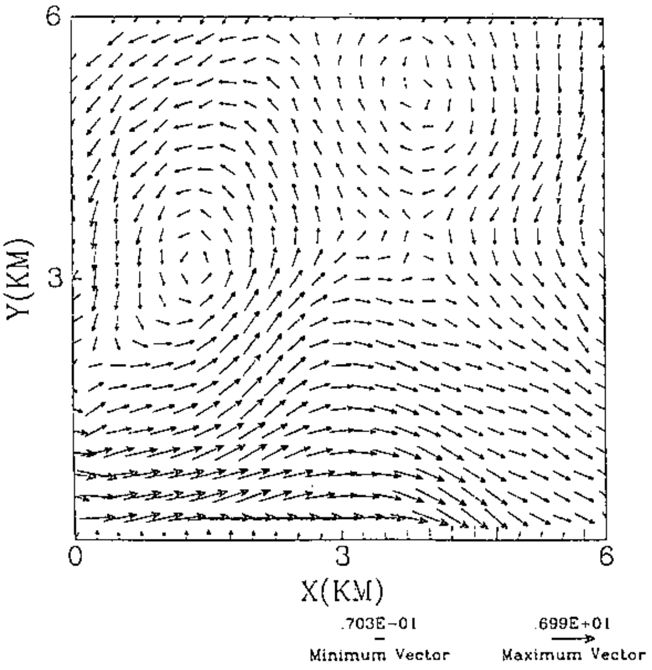

Fig. 5 Retrieved wind using 2 volume scans in Group B experiments.

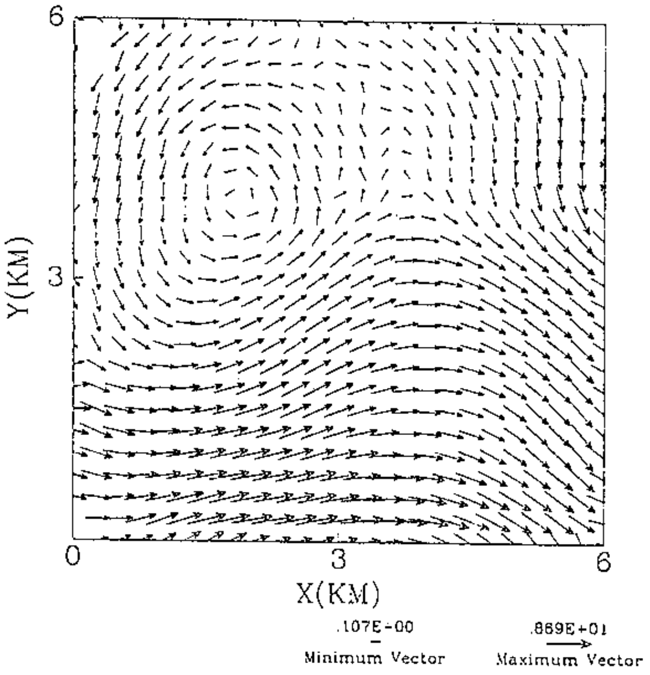

Fig. 6 Same as Fig. 5, but with 4 volume scans.

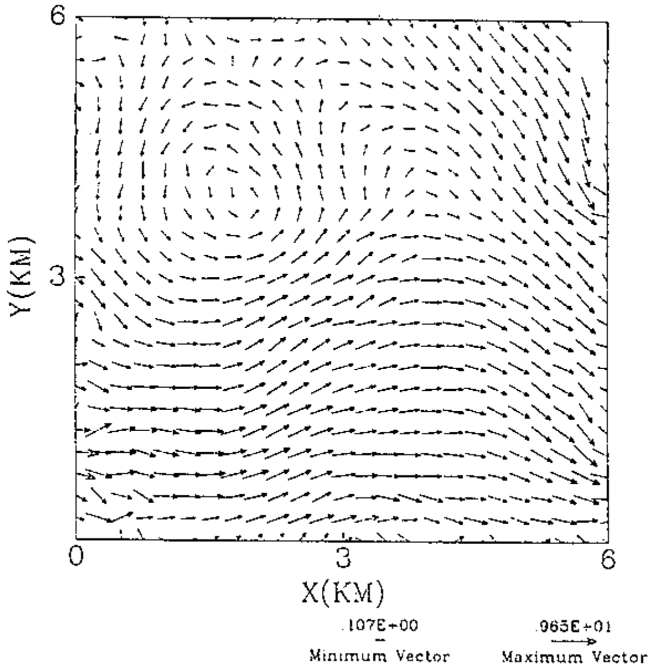

Fig. 7 Retrieved wind using reflectivity data only.